Series-parallel resistor network

Q. A 24 V battery drives $R_1 = 4\,\Omega$ in series with $R_2 = 12\,\Omega$ and $R_3 = 6\,\Omega$ in parallel. Find the current through each resistor.

1 Combine the parallel pair

FORMULA

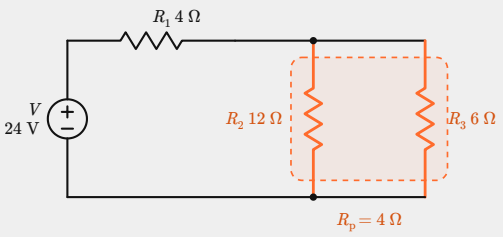
$$R_p = \frac{R_2 R_3}{R_2 + R_3}$$

WORK

R_2 and R_3 connect across the same two nodes, so they are in parallel.

$$R_p = \frac{12 \times 6}{12 + 6} = \frac{72}{18} = 4\,\Omega$$

DIAGRAM



TAKEAWAY

A parallel pair is always smaller than its smallest branch: $4\,\Omega < 6\,\Omega$.

2 Find the total current

FORMULA

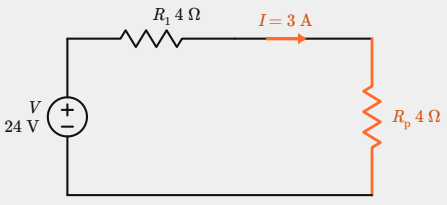
$$I = \frac{V}{R_T}$$

WORK

Now R_1 and R_p are in series: $R_T = R_1 + R_p = 4 + 4 = 8\,\Omega$.

$I = \dfrac{24\,\text{V}}{8\,\Omega} = 3\,\text{A}$. This current leaves the battery and flows through R_1.

DIAGRAM



TAKEAWAY

Reduce the network to one resistance first. Ohm's law then gives the source current in one line.

3 Split the battery voltage

FORMULA

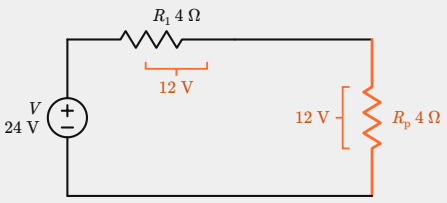
$$V = IR$$

WORK

$V_1 = 3\,\text{A} \times 4\,\Omega = 12\,\text{V}$ across R_1.

$V_p = 3\,\text{A} \times 4\,\Omega = 12\,\text{V}$ across the parallel pair. Check: $12 + 12 = 24\,\text{V}$.

DIAGRAM



TAKEAWAY

Equal resistances in series take equal shares of the source voltage.

4 Share the current between branches

FORMULA

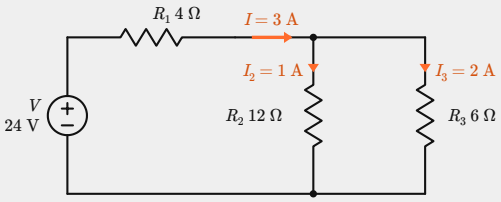
$$I_k = \frac{V_p}{R_k}$$

WORK

Both branches see $V_p = 12\,\text{V}$: $I_2 = 12/12 = 1\,\text{A}$ and $I_3 = 12/6 = 2\,\text{A}$.

Check with KCL: $I_2 + I_3 = 1 + 2 = 3\,\text{A}$, the current through R_1.

DIAGRAM



TAKEAWAY

Current favours the easier path. R_3 has half the resistance of R_2, so it carries twice the current.